Int. J. Solids Structures Vol. 18, No. 11, pp. 937–945, 1982
Printed in Great Britain.
0020-7683/82/110937-09$03.00/0

DYNAMIC RESPONSE OF VISCOELASTIC PLATES OF ARBITRARY SHAPE TO RAPID HEATING

D. HILL, J. MAZUMDAR and D. L. CLEMENTS
Department of Applied Mathematics, The University of Adelaide, South Australia

(*Received* 16 *September* 1981; *in revised form* 23 *March* 1982)

Abstract—The dynamic behaviour of viscoelastic plates of arbitrary shape subjected to elevated temperatures is examined. Using a method based upon the concept of isoamplitude contour lines in conjunction with isothermal contour lines on the surface of the plate, a simple general approach for the study of thermally induced vibrations of a viscoelastic plate is presented here. The resulting method of solution is applied to study the response of a viscoelastic plate in the form of a hollow elliptical annulus and a viscoelastic rectangular plate under a thermal shock at the centre. For rapidly applied heat inputs, an approximate analysis for its rapid estimation is also presented. Numerical calculations are carried out for various values of geometrical and mechanical parameters and the results are illustrated by graphs.

1. INTRODUCTION

Thermally induced vibrations of viscoelastic plates have been the subject of useful study in the recent past because of their practical importance in modern nuclear technology. Under high temperatures the material exhibits the phenomenon of creep and thus mechanical properties of plates must be time dependent as well as temperature dependent. In an earlier paper[1], the authors showed how a simplified approach can be made to study the dynamic response of viscoelastic plates at elevated temperatures and as an illustration of the procedure, the vibration of a viscoelastic circular plate under a thermal shock at the centre was discussed. In another paper[2], an indication was given as to how the Berger technique can be used to study the dynamic behaviour of certain viscoelastic plates. Some recent work done by Nagaya[3–5] on vibration analysis of viscoelastic plates without taking thermal effects into consideration is also worth mentioning.

In the present paper, the method in Ref. [1] is expanded to study the dynamic behaviour of a hollow elliptical plate subjected to a suddenly applied heat input, and a rectangular plate under a thermal shock at the centre. The theory developed here gives a closed-form solution for the time-history response of viscoelastic plates. The results should be useful to nuclear engineers.

2. BASIC EQUATIONS

By using the procedure outlined in Ref. [1], the dynamical equation for thermally induced vibrations of a viscoelastic plate at any time τ can be expressed in the following integro-differential equation form

$$D(p)\frac{\partial^3 w}{\partial u^3}\oint_{C_u} R_1\,\mathrm{d}s + D(p)\frac{\partial^2 w}{\partial u^2}\oint_{C_u} F_1\,\mathrm{d}s + D(p)\frac{\partial w}{\partial u}\oint_{C_u} G_1'\,\mathrm{d}s$$
$$- D(p)[1+\mu(p)]\alpha_T\oint_{C_u}\left(u_x\frac{\partial T}{\partial x} + u_y\frac{\partial T}{\partial y}\right)\frac{\mathrm{d}s}{\sqrt{t}}$$
$$+\int_{\Omega_u}\int \rho h\frac{\partial^2 w}{\partial \tau^2}\,\mathrm{d}\Omega = 0 \qquad (1)$$

where $p = (\partial/\partial\tau)$, $D(p)$ and $\mu(p)$ are viscoelastic operators corresponding, respectively, to the elastic constants D (flexural rigidity) and μ (Poisson's ratio) and α_T is the coefficient of linear thermal expansion of the material.

Here the deflection W is considered to be a suitable function of u and τ, where $u(x, y) =$ constant is the equation of isoamplitude contour lines. The expressions for R_1, F_1 and G_1' appearing in eqn (1) are listed in Ref. [6] and the expression for t given by $t = u_x^2 + u_y^2$.

The solution of the above equation can be found in the same way as for the free vibration problem as follows. Firstly, the eigenfunctions W_i and the eigenfrequencies λ_i of the corresponding elastic problem under homogeneous boundary conditions are determined. It is well-known that this eigenvalue problem is self-adjoint and hence the associated eigenfunctions form a complete set and are mutually orthogonal. The viscoelastic solution is then obtained by assuming that the deflection w and the temperature T appearing in the above equation can be expressed as a linear sum of the eigenfunctions W_i in the form

$$w = \sum_{i=1}^{\infty} g_i(\tau) W_i(u), \quad T = \sum_{i=1}^{\infty} T_i(\tau) W_i(u). \tag{2}$$

Since W_i is a solution of the corresponding elastic equation then, by using the heat conduction equation for a plate and the orthogonality condition of the eigenfuctions, one obtains from eqn (1) an infinite number of linear differential equations for the $g_j(\tau)$ as

$$\lambda_j^2 \frac{D(p)}{D} g_j(\tau) + \frac{d^2 g_j(\tau)}{d\tau^2} = -\frac{\alpha_T \sigma(1+\mu(p))}{kh} D(p) \frac{\partial T_j}{\partial \tau} \quad j = 1, 2, 3, \ldots \tag{3}$$

where σ is the specific heat and k is the coefficient of thermal conductivity of the plate.

3. APPLICATIONS

In order to assess the accuracy of the method, the following cases will now be discussed.

Case 1

A hollow elliptical plate bounded externally and internally by confocal ellipses as shown in Fig. 1.

Assume that the plate is initially at a uniform temperature T_0 throughout and the boundary temperatures are suddenly changed to zero temperature. With the semi-major and semi-minor axes of the elliptic boundaries of the plate being denoted by a, b and a_1, b_1, respectively, the two equations are

$$1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \text{ for the outer boundary } \Gamma_1$$

$$1 - \frac{x^2}{a_1^2} - \frac{y^2}{b_1^2} = 0 \text{ for the inner boundary } \Gamma_2, \tag{4}$$

with the similarity condition for the two ellipses

$$a_1/a = b_1/b = \beta, \quad 0 < \beta < 1. \tag{5}$$

If attention is restricted to symmetric modes of vibration, then the eigenfunctions for the corresponding elastic problem are given by [6]

$$W_j = A_1 J_0(k_j f) + A_2 Y_0(k_j f) + A_3 I_0(k_j f) + A_4 K_0(k_j f) \ldots \tag{6}$$

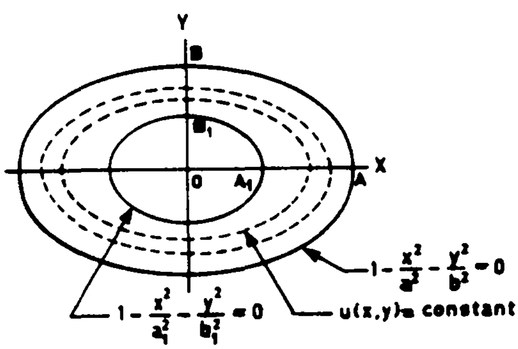

Fig. 1. Annular plate bounded by confocal ellipses.

where A_1, A_2, A_3 and A_4 are constants and f and k_j are given by

$$f^2 = 1 - u,\ u = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2},\ k_j^4 = \frac{8\rho h a^4 b^4 \lambda_j^4}{(3a^4 + 2a^2b^2 + 3b^4)}. \tag{7}$$

If the inner and the outer boundary of the plate are assumed to be either clamped or simply supported, then the required boundary conditions at the two edges are

$$w = 0,\ M_n = \gamma_1 \frac{\partial w}{\partial n} \text{ for the outer boundary } \Gamma_1$$

and

$$w = 0,\ M_n = \gamma_2 \frac{\partial w}{\partial n} \text{ for the inner boundary } \Gamma_2 \tag{8}$$

where γ_1 and γ_2 are spring constants. If $\gamma_1 = \gamma_2 = 0$, we obtain simply supported edges, and for $\gamma_1 = \gamma_2 = \infty$, we have clamping at the edges. The expressions for bending moment M_n and slope $(\partial w/\partial n)$ as given in [7] are

$$M_n = P\frac{\partial^2 w}{\partial u^2} + Q\frac{\partial w}{\partial u}, \tag{9}$$

$$\frac{\partial w}{\partial n} = -\sqrt{t}\frac{\partial w}{\partial u}. \tag{9}$$

Using the same technique of satisfying the boundary conditions by taking the mean values of P, Q and $\sqrt{t}$ as explained in [7], the boundary conditions (8) can be expressed in the new cariable f as

$$w = \frac{d^2 w}{df^2} + \left(\frac{\mu}{f} + \alpha\gamma_1\right)\frac{dw}{df} = 0 \text{ at the boundary } \Gamma_1 \text{ for } f = 1$$

and

$$w = \frac{d^2 w}{df^2} + \left(\frac{\mu}{f} + \alpha\gamma_2\right)\frac{dw}{df} = 0 \text{ at the inner boundary } \Gamma_2 \text{ for } f = \beta \tag{10}$$

where

$$\alpha = \frac{ab(a+b)}{D(a^2+b^2)}. \tag{11}$$

If use is made of the above four conditions expressed by eqn (10), the values of the constants $A_1, \ldots, A_4$ appearing in eqn (6) can be determined. The resultant form for W_j's thus becomes

$$W_j = \begin{bmatrix} J_0(k_j) & Y_0(k_j) & I_0(k_j) & K_0(k_j) \\ J_0(\beta k_j) & Y_0(\beta k_j) & I_0(\beta k_j) & K_0(\beta k_j) \\ k_j^2 J_0''(k_j) & k_j^2 Y_0''(k_j) & k_j^2 I_0''(k_j) & k_j^2 K_0''(k_j) \\ +\phi_1 k_j J_0'(k_j) & +\phi_1 k_j Y_0'(k_j) & +\phi_1 k_j I_0'(k_j) & +\phi_1 k_j K_0''(k_j) \\ J_0(k_j f) & Y_0(k_j f) & I_0(k_j f) & K_0(k_j f) \end{bmatrix} \tag{12}$$

where the k_j's satisfy the characteristic determinant

$$\begin{bmatrix} J_0(k_j) & Y_0(k_j) & I_0(k_j) & K_0(k_j) \\ J_0(\beta k_j) & Y_0(\beta k_j) & I_0(\beta k_j) & K_0(\beta k_j) \\ k_j^2 J_0''(k_j) & k_j^2 Y_0''(k_j) & k_j^2 I_0''(k_j) & k_j^2 K_0''(k_j) \\ \phi_1 k_j J_0'(k_j) & +\phi_1 k_j Y_0'(k_j) & +\phi_1 k_j I_0'(k_j) & +\phi_1 k_j K_0'(k_j) \\ k_j^2 J_0''(k_j) & k_j^2 Y_0''(k_j) & k_j^2 I_0''(k_j) & k_j^2 K_0''(k_j) \\ +\phi_2 k_j J_0'(k_j) & +\phi_2 k_j Y_0'(k_j) & +\phi_2 k_j I_0'(k_j) & +\phi_2 k_j K_0'(k_j) \end{bmatrix} = 0 \tag{13}$$

and where we have denoted

$$\mu + \alpha\gamma_1 = \phi_1, \ \frac{\mu}{\beta} + \alpha\gamma_2 = \phi_2. \tag{14}$$

Here primes denote differentiation with respect to the argument. With the eigenfunctions and eigenvalues thus having been determined, the next task is to analyse the effects of temperature in the region of the plate. The corresponding temperature distribution for this problem will be [8]

$$T = T_0\Pi\sum_{n=1}^{\infty} \frac{J_0(\omega_n\beta)}{J_0(\omega_n\beta) + J_0(\omega_n)} U_0(\omega_n f)\, e^{-(c^2(a^2+b^2)/2a^2b^2)\omega_n^2\tau} \tag{15}$$

where

$$U_0(\omega_n f) = J_0(\omega_n f)\,Y_0(\omega_n) - J_0(\omega_n)\,Y_0(\omega_n f) \tag{16}$$

and ω_n's are the roots of $U_0(\omega_n\beta) = 0$. Here c is a material constant depending on the specific heat and the thermal conductivity of the material.

Since the temperature field and the elastic eigenfunctions of the problem are now known, eqn (3) can now be solved for the time functions $g_j(\tau)$ for any particular viscoelastic material. This will be explained further in the next section.

Case 2

A rectangular plate under a thermal shock at its centre.

Assume the plate is subjected to an instantaneous point heat source at the centre, and beginning from an instant $\tau = 0$ the edges are kept at zero temperature. The solution to this thermal shock problem is pertinent to many practical situations in welding engineering as well as modern nuclear technology.

For a rectangular plate with sides of length a and b (see Fig. 2) a good approximate equation of isoamplitude contour lines under symmetric modes of vibration can reasonably be considered as [7]

$$u(x, y) = \left(1 - \frac{4x^2}{a^2}\right)\left(1 - \frac{4y^2}{b^2}\right) \tag{17}$$

which in fact represents the equation of the boundary. The eigenfunctions W_j in this case are approximated as finite power series in f in the form

$$W_j(f) = \sum_{i=0}^{m} C_{ij} f^i \tag{18}$$

where the coefficients C_{ij} are calculated by the usual method of collocation after satisfying the required boundary conditions. In this study, seven terms of the power series solutions have been used. Calculations of more coefficients showed that the effects of their inclusion in the series is negligible.

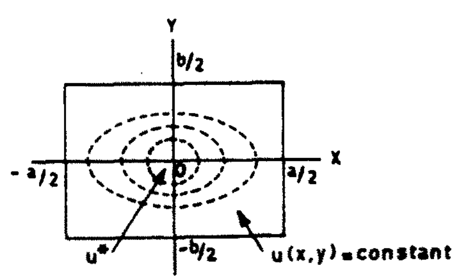

Fig. 2. Rectangular plate.

In order to discuss the effects of temperature under a thermal shock at the centre of the plate, one is required to solve the corresponding heat conduction equation[8] as given by

$$\frac{\mathrm{d}\bar{T}}{\mathrm{d}u}\oint_{C_u} \nabla t\,\mathrm{d}s + s/c^2\int_{\Omega_u}\int \bar{T}\,\mathrm{d}\Omega = \frac{1}{c^2}\int_{\Omega_u}\int T_0\,\mathrm{d}\Omega \tag{19}$$

where the Laplace transform of $T(u, \tau)$ is denoted by $\bar{T}(u, s)$ and $T_0(u)$ is the initial temperature which obviously is zero in the present problem.

Differentiating the above equation with respect to u making use of the mean value theorem for evaluating the resulting contour integrals, one finally arrives at the following second order differential equation in the variable f

$$\frac{\mathrm{d}^2\bar{T}}{\mathrm{d}f^2} + \frac{1}{f}\frac{\mathrm{d}\bar{T}}{\mathrm{d}f} - \omega_2\bar{T} = 0 \tag{20}$$

where $\omega^2 = abs/4c^2$. The above equation is recognised as a form of the modified Bessel's equation of order zero which has it complete solution as

$$\bar{T}(f, s) = A(s)I_0(\omega f) + B(s)K_0(\omega f) \tag{21}$$

where I_0 and K_0 are modified Bessel functions. The unknown coefficients A and B are to be obtained from initial and boundary conditions of the problem, which are as follows:

(i) $T = 0$ for $\tau = 0$, everywhere in the region of the plate except at the centre $u = 1$,

(ii) $$\lim_{u\to 0}\oint_{C_u}\frac{\partial T}{\partial n}\,\mathrm{d}s = -\frac{q}{c^2}\delta(\tau),$$

(iii) $$T = 0 \text{ on the boundary } u = 0,\ \tau \geq 0, \tag{22}$$

where $\delta(\tau)$ is the Dirac delta function and q is the strength of a point heat source. Taking the Laplace transforms of boundary conditions (ii) and (iii) above, one finally obtains, in term of the variable f,

$$\lim_{f\to 0} f\frac{\mathrm{d}\bar{T}}{\mathrm{d}f} = -q/\epsilon\pi c^2,\ \bar{T}(f) = 0 \text{ for } f = 1 \tag{23}$$

where

$$\epsilon = (a+b)^2/2ab,$$

on using these conditions in eqn (21), the constants A and B are obtained as

$$A = -\frac{q}{\epsilon\pi c^2}\frac{K_0(\omega)}{I_0(\omega)},\ B = \frac{q}{\epsilon\pi c^2}. \tag{24}$$

Hence the solution $\bar{T}(f, s)$ can be written as

$$\bar{T} = \frac{q}{\epsilon\pi c^2}\left[\frac{I_0(\omega)K_0(\omega f) - K_0(\omega)I_0(\omega f)}{I_0(\omega)}\right]. \tag{25}$$

Using the inversion theorem[9], the solution $T(f, \tau)$ is found to be

$$T(f, \tau) = \frac{16q}{(a+b)^2}\sum_{n=1}^{\infty}\frac{J_0(\omega_n f)}{J_1^2(\omega_n)}\,\mathrm{e}^{-(4c^2/ab)\omega_n^2\tau}, \tag{26}$$

where ω_n's are the roots of $J_0(\omega_n) = 0$, and J_0 and J_1 are Bessel functions of the first kind.

Thus the temperature distribution at a generic point (x, y) of the plate can be obtained using the above formula.

A similar approach for the case of perfectly general plates of visco-elastic materials can be used once the contour equations for such plates are known. Many such contour equations have previously been discussed in the literature, for example Equilateral Triangle [8], Parabola [7], Semicircle [12] and for general shaped plates [13].

4. VISCOELASTIC MODEL

The viscoelastic model adopted in this study is one which behaves elastically in dilatation and has the viscoelastic behaviour of the Kelvin model in shear. This particular model is chosen primarily for the sake of illustration and is considered to be a good representation of viscoelastic behaviour of commonly used materials in nuclear technology. In this model, the operator $D(p)$ takes the following form

$$D(p)=\frac{h^3(E_1+\eta p)(6K+E_1+\eta p)}{12(3K+2E_1+2\eta p)} \tag{27}$$

where η, K and E_1 are material constants which can be expressed in terms of Young's modulus and Poisson's ratio in the form

$$K=\frac{E}{3(1-2\mu)},\quad E_1=\frac{E}{1+\mu}.$$

Substituting the above form for $D(p)$ in eqn (3) one obtains a third ordinary differential equation,

$$\{\lambda_j^2 D_1(p)+Dp^2D_2(p)\}g_j(\tau)=-ApD_1(p)T_j(\tau) \tag{28}$$

where $A=\alpha_T\sigma(1+\mu)D/kh$; μ is considered constant

$$D_1(p)=h^3(E_1+\eta p)(6K+E_1+\eta p)/12,$$

$$D_2(p)=(3K+2E_1+2\eta p). \tag{29}$$

In order to solve eqn (28) by the Laplace transform method, one needs to know the corresponding initial conditions. By using the method developed by Deleeuw [10] for determining initial conditions for the time equation of a viscoelastic plate, one obtains the required initial conditions for the case where the plate is constrained in its deformed position for a long time prior to its release and consequently all viscous effects have decayed to zero by the time of release. That is, the internal forces within the plate are initially elastic ($\eta=0$). Hence the initial conditions for the time equation are

$$g_j(0)=0,\quad g_j'(0)=-AT_j(0),\ \text{and}\ \ g_j''(0)=-AT_j'(0), \tag{30}$$

where the prime denotes differentiation with respect to τ. Equation (28) can be written in the polynomial form as

$$\left\{\sum_{n=0}^{3}a_np^n\right\}g_j(\tau)=-A\left\{\sum_{n=1}^{3}b_np^n\right\}T_j(\tau), \tag{31}$$

where the coefficients a_n and b_n contain only material constants. The above equation can now be solved with the aid of the Laplace transform method with initial conditions (30). This yields

$$\begin{aligned}P_j(s)\bar{g}_j(s)=-A[&\{b_1s+b_2s^2+b_3s^3\}\bar{T}_j(s)\\&-\{(b_1-a_2)+(b_2-a_3)s+b_3s^2\}T_j(0)\\&-\{(b_2-a_3)+b_3s\}T_j'(0)-b_3T_j''(0)],\end{aligned} \tag{32}$$

where

$$P_j(s) = \sum_{n=0}^{3} a_n s^n, \tag{33}$$

and the bars denote Laplace transforms, s being the Laplace transform parameter.

The solution to eqn (32) can be obtained by inverse Laplace transformation when the functions $T_j(\tau)$ are known. The functions $T_j(\tau)$ can however be obtained from eqn (2) by making use of the orthogonality conditions of the eigenfunctions $w_j(\tau)$ given by

$$\int_{f^*}^{1} f w_i w_j \, df = \delta_{ij} B_i, \tag{34}$$

where δ_{ij} is the Kronecker delta and f^* is the value of f corresponding to u_{max}. Thus, for the Case 1 of hollow elliptical plate one obtains from eqn (15)

$$T_j(\tau) = \frac{T_0 \Pi}{B_j^E} \sum_{n=1}^{\infty} \frac{J_0(\omega_n \beta)}{J_0(\omega_n \beta) + J_0(\omega_n)} \left\{ \int_\beta^1 f w_j^E(f) U_0(\omega_n f) \, df \right\} \times e^{-(c^2(a^2+b^2)/2a^2b^2)\omega_n^2 \tau}, \tag{35}$$

while for the Case 2 of rectangular plate one obtains from eqn (26)

$$T_j(\tau) = \frac{16q}{(a+b)^2 B_j^R} \sum_{n=1}^{\infty} \left\{ \int_0^1 f w_j^R(f) J_0(\omega_n f) \, df \right\} e^{-(4c^2/ab)\omega_n^2 \tau} \Big/ J_1^2(\omega_n) \tag{36}$$

where B_j^E and B_j^R denote respectively the corresponding value obtained from eqn (34) for the elliptical and rectangular plates and $w_j^E(f)$ and $w_j^R(f)$ are the corresponding eigenfunctions.

5. NUMERICAL RESULTS

Numerical analysis is carried out with a view to looking at the various effects of geometrical and mechanical parameters on the vibrational behaviour of plates. Figure 3 shows the effect of

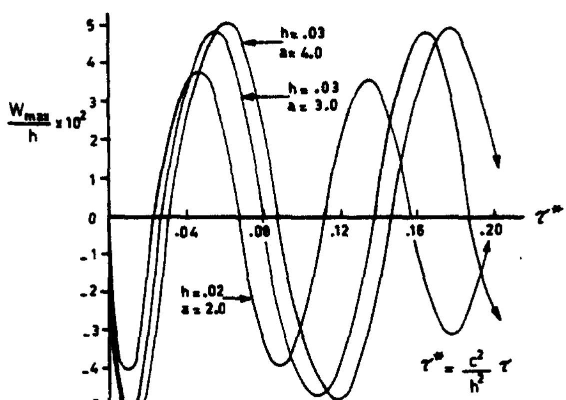

Fig. 3. Plot of non-dimensional maximum deflection vs non-dimensional time for the elliptical plate.

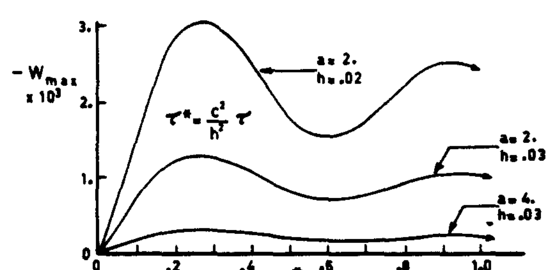

Fig. 4. Plot of non-dimensional maximum deflection vs non-dimensional time for the rectangular plate.

Table 1. Values of w for various values of $f(=\sqrt{(1-u)})$ and $\beta\left(=\frac{a_1}{a}=\frac{b_1}{b}\right)$

f / β	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.3	0.000	−0.0001	−0.0001	−0.0002	−0.0002	−0.0001	−0.0000	0.0000
0.5	—	—	0.0000	−0.0007	−0.0016	−0.0015	−0.0006	0.0000
0.7	—	—	—	—	0.0000	−0.0224	−0.0212	0.0000

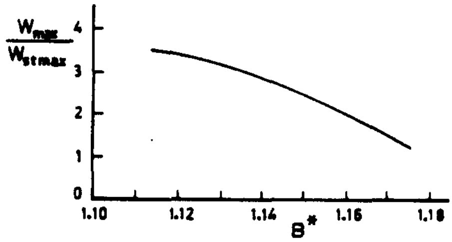

Fig. 5. Plot of W_{max}/W_{stmax} vs B^* for the elliptical plate.

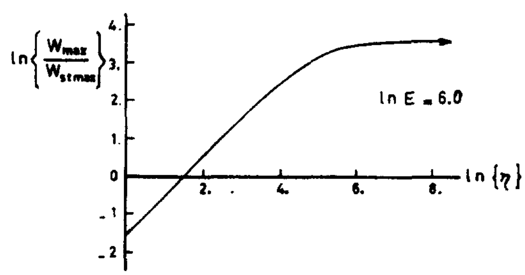

Fig. 6. Plot of $\ln\left\{\frac{W_{max}}{W_{stmax}}\right\}$ vs $\ln\{\eta\}$ for the elliptical plate.

altering the values of the aspect ratio a/b and the thickness h, respectively, in the case of the elliptical plate. Figure 4 shows the effects of altering these same parameters for the rectangular plate. With the annular elliptic plate the effect of altering the size of the annular hole can also be studied (see Table 1). It can be seen that as the value of the ratio parameter β increases, the amplitude of the vibration increases markedly.

In Fig. 5 a plot of the ratio of maximum total to maximum static deflection vs a non-dimensional time parameter is given for the elliptical plate. The static deflection W_{st} is obtained by solving eqn (1) while disregarding inertia and damping. The non-dimensional time parameter B^* is a ratio of characteristic thermal time to characteristic mechanical time (following the line of Jadeja and Loo [11]) as is given by

$$B^*=\frac{h}{ac}\left\{\frac{D}{\rho h}\right\}^{1/4}.$$

Variation in B^* can hence be obtained by altering h, a, c, ρ, E or μ. For Fig. 5 different values of B^* were obtained by altering μ, and the plate was taken to be almost elastic, i.e. a very small value of η was chosen. As η is allowed to increase, the ratio w_{max}/w_{stmax} increases quickly (Fig. 6). Thus for plates that are almost elastic (small η) the inertia and damping terms cannot be ignored but as the plate becomes successively more "viscous" the inertia and damping terms have successively less influence on the total deflection. The geometry of a plate and the type of heating also effect its mode of vibration as evidenced by comparing Figs. 3 and 4. All numerical results were calculated while taking both plates to be clamped on all of their boundaries.

In essence, the particular model that one chooses for a study of visco-elastic material behavior greatly affects the resultant vibrations and this will be studied in greater detail in a subsequent paper.

Acknowledgement—The authors wish to acknowledge the partial support of the Australian Research Grants Committee who sponsored part of this investigation under grant F78/15085.

REFERENCES

1. J. Mazumdar, D. Hill and D. L. Clements, Thermally induced vibrations of a viscoelastic plate. *J. Sound Vib.* **73**, 31–39 (1980).
2. R. Jones, J. Mazumdar and Y. K. Cheung, Vibration and buckling of plates at elevated temperatures. *Int. J. Solids Structures* **16**, 61–70 (1980).
3. K. Nagaya, Vibrations and dynamic response of viscoelastic plates on non-periodic elastic supports. *J. Eng. Industry, Trans. ASME, Ser. B.* **99**, 404–409 (1977).
4. K. Nagaya, Dynamics of viscoelastic plate with curved boundaries of arbitrary shape. *ASME J. Appl. Mech.* **45**, 629–635 (1978).
5. K. Nagaya, Transverse vibration of a rectangular plate with an eccentric circular inner boundary. *Int. J. Solids Structures* **16**, 1007–1016 (1980).
6. J. S. Hewitt and J. Mazumdar, Vibration of viscoelastic plates under transverse load by the method of constant deflection contours. *J. Sound Vib.* **33**, 319–333 (1974).
7. J. Mazumdar, A method of solving problems of elastic plates of arbitrary shape. *J. Australian Mathematical Soc.* **11**, 95–112 (1970).
8. J. Mazumdar, A method for the study of transient heat conduction in plates of arbitrary cross-section. *Nuclear Engng Design* **31**, 383–390 (1974).
9. H. S. Carslaw and J. C. Jaeger, *Operational Methods in Applied Mathematics*, 2nd Edn. Oxford University Press, London (1948).
10. S. L. Deleeuw, Behaviour of viscoelastic plates under the action of in-plane forces. Ph.D. Thesis, Michigan State University (1961).
11. N. O. Jadeja and Ta-Cheng Loo, Heat induced vibration of a rectangular plate. *ASME Trans., Series B, J. Engng for Industry* **96**, 1015–1021.
12. R. Jones and J. Mazumdar, A more exact solution for the bending of a semicircular elastic plate by the method of constant deflection contours. *Proc. 3rd Australasian Conf. on Mech. of Structures and Materials*, New Zealand, Aug. 1971.
13. R. Jones, J. Mazumdar and Fu-Pen Chiang, Further studies in the application of the method of constant deflection lines to plate bending problems. *Int. J. Engng Sci.* **13**, 423–443 (1975).